Opening Statistics and Match Play for Backgammon Games

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Abstract. Players of complex board games like backgammon, chess and go, were always wondering what the best opening moves for their favourite game are. In the last decade, computer analysis has offered more insight to many opening variations. This is especially true for backgammon, where computer rollouts have radically changed the way human experts play the opening. In this paper we use Palamedes, the winner of the latest computer backgammon Olympiad, to make the first ever computer assisted analysis of the opening rolls for the backgammon variants Portes, Plakoto and Fevga (collectively called Tavli in Greece). We then use these results to build effective match strategies for each game variant.

Keywords: Monte Carlo, Game Statistics, Match play, Backgammon, Plakoto, Fevga

1 Introduction

Backgammon is a perfect information, turn-taking game of two players, where the outcome is influenced both from skill and the roll of the dice. At each turn, the available candidate moves are computed according to the roll of two six-sided dice, resulting in 21 possible rolls. Standard backgammon opening rolls have been thoroughly analyzed in [6]. To the best of our knowledge, this kind of analysis has not been made in other backgammon variants. In this paper we attempt to computationally analyze the opening rolls of the backgammon variants Portes, Plakoto and Fevga, using our Palamedes bot[1]. We then use these results to extract useful statistical information about the games.

Our methodology is similar to the one used in [6]: The most promising continuations after each roll are analyzed using rollout analysis, a Monte Carlo method that is commonly used in backgammon. Starting from the resulting position after each candidate move, a fixed number of games is played until a terminal position is reached. Counting the results of these games we can finally get the probabilities of single wins

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[1] Palamedes can be freely downloaded from http://ai.uom.gr/nikpapa/Palamedes

(WS), double wins (WD), single losses (LS) and double losses (LD). Bases on these probabilities, we can then compute the estimated equity of each position using the following equation:

$$E = WS - LS + 2 * (WD - LD) \tag{1}$$

This kind of evaluation is considered to offer accurate results in backgammon, despite the fact that the move selection algorithm of the rollout phase is not so strong in terms of performance [12]. Rollouts can also be truncated, which means that they could stop after a fixed amount of plies (instead of going till the end of the game) and average together the estimates of the resulting positions, with a negligible change in their estimates.

The rest of the paper is structured as follows: First we briefly discuss the related work and the three backgammon variants, then we present our experimental results, we discuss them and, finally, we conclude the paper and identify some challenges for future research.

2 Background

2.1 Related work

Monte Carlo methods have recently gained increased interest by game AI researchers due to the success of the MCTS algorithm in the game of Go [3, 5], as well as in other games [4, 7, 14]. In MCTS and it's most popular variant UCT, rollouts (usually called playouts) are used to simulate a trajectory, whereas its outcome is used to build and update a tree from the starting position. The rollouts can be random or based on heuristics. Recent advances in computer Go indicate that adding domain knowledge in the rollouts is critical for producing state of the art performance.

More relevant to our setup are the MCTS variants that attempt to solve the game-theoritic value of positions. MCTS-Solver [13] and Monte Carlo Proof-Number Search (MC-PNS) [10] are two such algorithms that are based on Proof Number Search [1]. Score bounded MCTS [2] is another technique for proving the game theoretic value of games of multiple outcomes. However, all three algorithms are suitable for determinist games only. To the best of our knowledge we are not aware of similar enhancements for non-deterministic, perfect information games with chance nodes such as backgammon.

2.2 Rules of backgammon variants examined

Most backgammon variants are usually conducted in a board containing 24 *points* divided in 4 quadrants of 6 points each. Each player starts the game with a number of checkers or stones at his disposal (usually 15) placed in a fixed starting position. The players take turns playing their checkers forward using an element of chance in the form of two six-sided dice. The checkers can be moved only to an *open point* according to the game rules. When all checkers of a player are inside his last quadrant of the board (called his *home* board), he can start taking them out of the board; this is called bearing off. The player that clears all his checkers first is the winner of the game.

At the end of the game, if the losing player has borne off at least one checker, he loses only one point. However, if the loser has *not* borne off any of his checkers, he loses a double game or *gammon* and two points. In standard backgammon, if the loser has not borne off any of his checkers and still has a checker in the winner's home board, he loses a triple game or *backgammon* and three points.

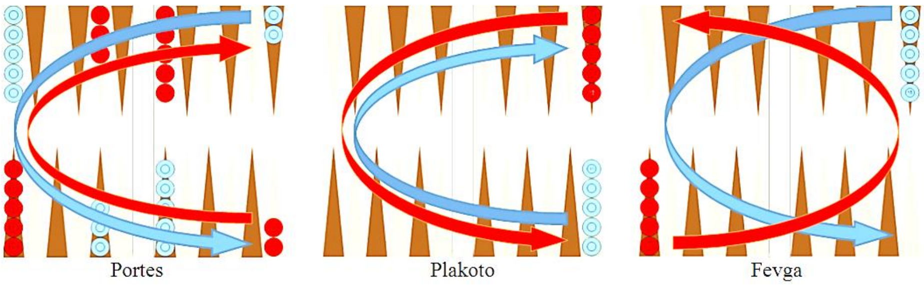

Fig. 1. Starting position and direction of movement for Tavli games.

There are hundreds of backgammon variants played around the world, however most of them can be classified into three categories, according to the rule that defines a player's *made point*, that is a point where only the player that has "made" it can move into:

- The *hitting games* (e.g. Backgammon, Portes, Acey-deucey), where players can "hit" lone checkers of the opponent, placing them on the *bar*, forcing the opponent to re-enter them in his home board before playing any other move. A *made point* in this type of games is a point containing two or more checkers.
- The *pinning games* (e.g. Plakoto, Tapa), where players can "pin" lone checkers of the opponent, thus preventing the movement of the pinned checker. A made point in this type of games is a point where two or more checkers of the same player exist or one checker that has pinned an opponent checker.
- The *running* games (e.g Fevga, Narde, Gul-bara), where no pin or hit is possible. A single checker in a point constitutes a "made point". In this type of games, movement of checkers is in the same direction for both players (Fig. 1, right), further complicating matters.

Fig. 1 shows the starting positions and the direction of checker movement for the three variants that we examine in this paper. As can be easily seen, Plakoto and Fevga have starting positions where all player checkers are stacked in their first point. On the other hand, Portes (and her almost twin backgammon) have a special crafted starting position. We examine the importance of the starting position later in the discussion section.

The rules mentioned above are the most important for the variants Portes, Plakoto and Fevga that we examine in this paper. There are some other details about the rules of Plakoto and Fevga that can be found in [2]. Portes is identical to backgammon, with the following changes: a) No cube[2] is used; b) no triple wins (backgammons); and c) a double roll is allowed in the first move.

2.3 Match play and Tavli

Backgammon games can be played individually, in which case they are called *money games*; however, more often they are played in a match, where each player accumulates points (one point for single wins, two points for double wins) until a player reaches a predefined number of points. In Greece, the most popular way of playing backgammon games is a Tavli match (Fig. 2), where Portes, Plakoto and Fevga are played one after the other, until a player reaches five or seven points.

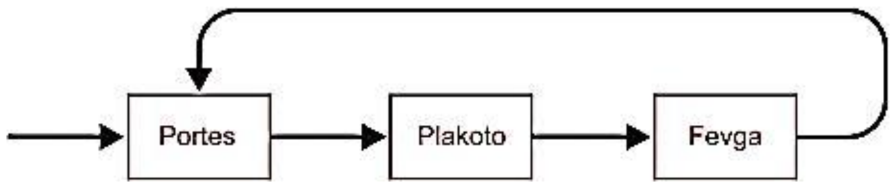

Fig. 2. Flow of a Tavli match. (Source: http://www.bkgm.com/variants/Tavli.html)

2.4 Palamedes

All the experiments in this paper are made using our latest Palamedes bot. Palamedes, originally started as a project aimed at developing expert-playing bots at Tavli games, currently supports Portes, Plakoto, Fevga, Narde, Hypergammon, Nackgammon, Takhteh, with more variants planned for the future. At the core of the evaluation function of each game is a Neural Network trained by TD(λ) [11] and millions of self-play games. The training procedure that we used [8, 9] is inspired by the early successes of TD-Gammon in backgammon [12]. Palamedes is the current world champion in computer backgammon, after taking the first place at the latest Backgammon Computer Olympiad held in Tilburg, Netherlands, 2011.

3 Experimental setup and results

We used our latest and best Neural Networks (NN) game evaluation functions for selecting each move on the rollouts. For Portes we used Portes_ACG13 NN, for Plakoto we used Plakoto5, and for Fevga we used Fevga6 [8]. The rollouts were performed using 1-ply playing mode, which means that Palamedes looked ahead only at the current roll for each play during the rollouts, selecting the best play of each trial. After the opening roll we rolled out the five most promising candidate moves (selected using 2-ply evaluation), using 100,000 games per position. The standard error of

[2] When the doubling cube is used the stakes of the game can be increased by the players. Using the cube speeds up match play and provides an added dimension for strategy.

the estimated equity E (1) when performing this number of trials is less than 0.02. Rollouts were performed using **cubeless untruncated money play**. *Cubeless* means that games are played without a doubling cube. *Untruncated* means that rollouts were played out until the end of the game. *Money play* means that each game is played individually and not as a part of a match.

Opening rolls were split in two groups, single and double, in order to shed more light into the effect of rolling a double at the start of the game. This is most useful in standard backgammon, which does not allow a double opening roll like the Portes variant does. The move selected for each roll was picked as the best after rolling out the most promising candidate moves available. These figures were constructed by singling out the move with best equity after each roll. The actual moves selected can be seen in Appendix B.

Figs. 3-5 summarize the results for each roll and game variant and compares the games. All numbers shown are with regard to the first player making the move. Averages of all single rolls are marked with the word ‘SINGLE’. Averages of all double rolls are marked with the word ‘DOUBLE’. Finally the word ‘ALL’ is the weighted (according to the probability of each roll) average of all 21 rolls.

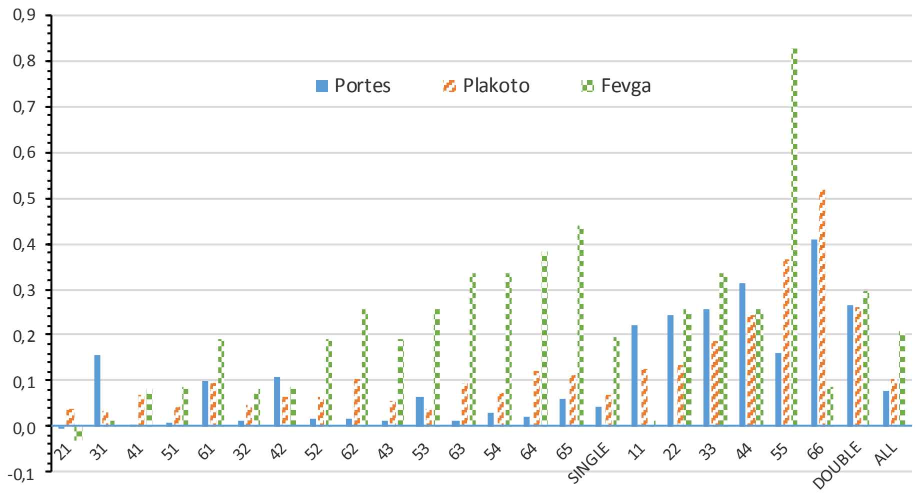

Fig. 3. Comparison of estimated equity of all opening rolls

In Fig. 3 the estimated equity of all opening moves for all games is presented. The starting roll with the greatest equity is by far the 55 in Fevga, while the least useful roll is the 21 in Fevga.

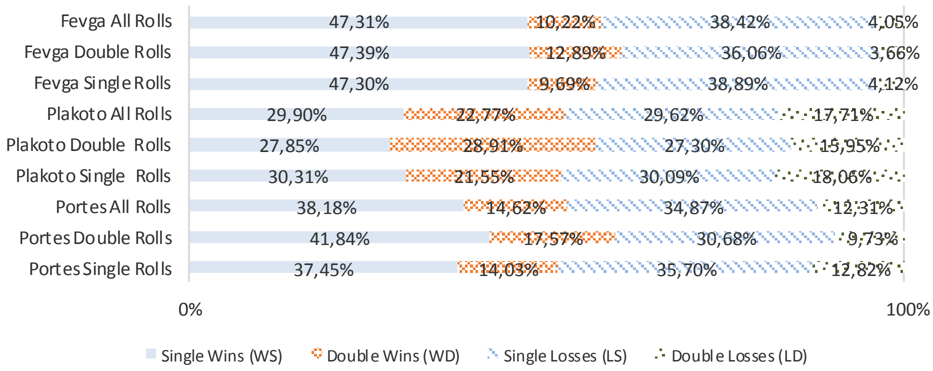

Fig. 4. Expected outcome (%) of the first player

Fig. 4 summarizes the outcome of all rolls to produce the expected result of the first player. From this figure we can derive the percentage of games that result in doubles, also called "gammon rate", by adding WD and LD (Table 1).

Table 1. Gammon rates of Tavli variants

Variant	Gammon Rate
Portes	26.85%
Plakoto	40.48%
Fevga	14.27%

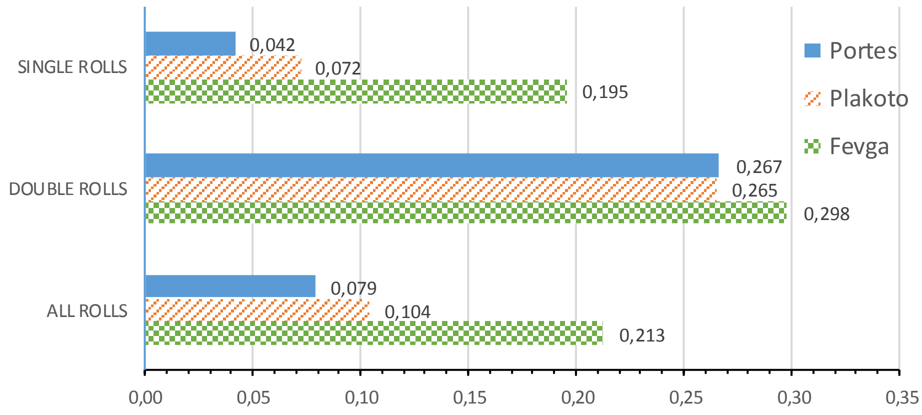

Fig. 5. Total estimated equity of the first player

Perhaps the most interesting result of this study is the total estimated equity of the first player shown in Fig. 5. Ideally, a perfectly designed backgammon game would give zero equity to the first player. This would mean that the opening roll does not favor one player over the other. Our study shows that the “best” variant in that regard is Portes, closely followed by Plakoto. On the other hand, Fevga gives a significant advantage to the first player.

4 Discussion

This section discusses and compares the results of the three games to each other, as well as to previous similar studies. We also attempt to explain some of the results found from a strategic point of view.

4.1 Portes

The results for the single rolls of the Portes variant are very similar to a previous study on standard backgammon openings [6]. In that study, the rollouts were performed by GnuBG[3], a very strong open source backgammon program at a 2-ply depth. The estimated equity of all single rolls in [6] is 0.039, ours is 0.042. Almost all best opening moves coincide with our best selected moves. The gammon rate is estimated in [6] at 27.6%. If we count the backgammons, which according to Portes rules are counted as gammons, this rate is increased to 28.8%. Our results estimate this at a more modest 26.9%, almost a 2% difference. We give two possible explanations for this behavior: a) 1-ply rollouts are not accurate enough, and b) the playing style of Palamedes is more conservative compared to that of GnuBG, resulting in somewhat fewer gammons.

Since the analysis of the single opening rolls is nothing new, we concentrate the discussion around the effect of the double rolls. The inclusion of doubles in the opening roll gives more advantage to the first player. The average equity of all double rolls is 0.267 (Fig. 5), six times larger than the equity of the single rolls. This was expected, since a) doubles usually result in more distance travelled than the average single roll, and b) even small doubles like 11 give the opportunity to construct strategically made points without risking getting hit by the opponent. The best double roll is 66 with 0.41 equity; even the worst double roll (11, E=0.22) is better than the best single roll (31, E=0.16). The effect of doubles can be seen in the weighted average of all rolls (Fig. 5, E=0.079), which is almost twice that of the single rolls.

4.2 Plakoto

Plakoto results, compared to the other games, demonstrate an increased gammon rate. 41% of Plakoto games are won as doubles, 14% more than the rate we calculated in Portes. This rate can be explained by the strategic strength of pinning an opponent checker inside his home board. It is well known that this kind of pin can result in

[3] http://www.gnubg.org

double games because if the pinning player manages not to get pinned himself, he can place his checkers in such a way that during bear off most of his pieces will be borne off before the pinning checker is unpinned. This places the pinned player at a great disadvantage, because usually he does not have enough time to return the last checker to his home board and avoid the double loss. Of course, one can play a very conservative game and avoid leaving lone checkers in his home board at all costs. However, this can lead to other problems: building large stacks of checkers, that are extremely inflexible and also minimize the chances of hitting lone checkers of the opponent. For this reason, Palamedes and most expert players prefer a "restrained aggressive approach" during the opening, leaving some lone checkers open when there is a small chance that the opponent can pin them. This strategy however, inevitably falls victim to a lucky pinning roll by the opponent, which may be enough to result in a double loss. This reasoning strongly suggests that the starting position of Plakoto (Fig. 1, middle) greatly influences the gammon rate and the equity of the first player.

In order to test the hypothesis above, we made another experiment changing the starting position: Instead of having all 15 checkers at the starting point, the checkers are distributed evenly in the first three points. This variant is known in some regions as the *Tapa* variant and we will use this naming also in this paper. The starting position of Tapa makes the pinning of checkers inside the home board during the opening more difficult, because the players can construct made points more easily during the start of the game. We used the same methodology and the same Neural Network (Plakoto-5) for the rollouts. Even if this NN was not trained for this specific variant, we believe that it is sufficient to produce strong play, because the type of positions resulting from a Tapa game are well within the range of positions the NN has seen during self-play training[4].

Table 2. Comparison of Tapa and Plakoto estimated results for the first player

Variant	**WS (%)**	**WD (%)**	**LS (%)**	**LD (%)**	**EQUITY**	**GAMMON RATE (%)**
Plakoto	29.89	22.77	29.62	17.71	0.104	40.48
Tapa	37.40	13.12	37.91	11.55	0.026	24.67

The results of the Tapa experiment (Table 2) confirm our hypothesis. The gammon rate is reduced from 40.48 to 24.67%. Also the equity of the first player is reduced to 0.026, which is even lower than the equity of the single rolls in the Portes variant (Fig. 5).

Another notable point that can be seen in Table 2 is that the first player wins about the same amount of single games as the second player (29.9% vs 29.6%). Conse-

[4] The opposite situation could be problematic: a Tapa trained NN may not evaluate correctly Plakoto's opening positions with early home board pins in points 2 and 3, because this kind of experience would have been extremely rare in its self-play training.

quently all the advantage that the first player has can be attributed to the difference in double games won which is 22.77% compared to 17.71% of the second player.

4.3 Fevga

The first interesting result in the Fevga experiments is that the expected equity of the first player (0.213) is the highest amongst all games examined, more than twice that of the Plakoto (0.104). Winning 57.5% of the games gives the player who plays first a distinctive advantage. Fevga also has the roll with the most gained equity in all games, the 55 roll at 0.84 equity. We also observe that all high sum rolls (e.g. 63, 64, 65) give very high equity for the first player, with 65 (E=0.44), even surpassing the best Portes roll (66, E=0.41). However, unlike the two other variants, doubles do not increase the equity of the single rolls that much (from 0.19 to 0.21). This can be attributed to the fact that apart form 55, the other two large doubles (44 and 66) that typically have increased equity, have a reduced effect because of Fevga's starting rule [2]. Overall, we note that the further the starting checker is able to move during the first roll, the better the chances are for the first player. This observation fully justifies the name of the game ('Fevga' means 'run' in Greek).

Another surprising observation is that the gammon rate (14.27%) is very low compared to the other variants. The greatest factor that affects this statistic is the very small chance of the second player winning a double game. With 4.05% the second player wins less than half doubles that the first player does (10.22%).

5 Match Play

In this section we show how we can use the statistics from Table 1 to construct effective match strategies for Tavli variants. When playing a match, the goal of the players is to win the match and not to maximize their expected reward at the individual games. For this reason all strong backgammon programs select the best move by approximating the Match Winning Chance (MWC) at each move selection. We present a simple method, similar to the one used in backgammon, for approximating MWC, using the estimates of the NN evaluations and the gammon rate computed in Table 1. For simplicity, we examine only matches of the same game type where the player that starts each game is determined randomly.

First, we build a table estimating MWC before the start of the game for all possible score differences during the course of the match. In the most simple case, that is, when the score is tied, the players have the same chance of winning the match. The table is calculated using the following recursive definition:

$$mwc(A,B) = S * mwc(A-1, B) + D * mwc(A-2, B) + S * mwc(A, B-1) + D * mwc(A, B-2) \quad (2)$$

where A is the remaining points left for player A to win the match, B is the remaining points left for player B to win the match, mwc(A,B) is the table entry specifying the probability of winning the match for the A player when the current score is A points away – B points away, S is the probability each player has of winning a single game

(= (1 - gammon rate) / 2), D is the probability each player has of winning a double game (=gammon rate / 2). Appendix A show the tables computed with this method for the games Portes, Plakoto, Fevga and match away scores up to 9.

Finally, for move selection, a similar equation is used for determining the MWC of each move:

$$MWC = WS * mwc(A\text{-}1, B) + WD * mwc(A\text{-}2, B) + LS * mwc(A, B\text{-}1) + LD * mwc(A, B\text{-}2) \quad (3)$$

where WS, WD, LS and LD are the output estimations of our neural network evaluation function.

5.1 Experiments in Match Play

In order to test the above method, we made an experiment playing 10000 5-point matches in the three variants examined where one player uses the “match” strategy and the other player uses the “money play” strategy that tries to maximize the value of each individual games. The match started half the time by the “match” player and the other half by the “money” player. The results along with some useful statistics that we stored during the course of the matches are shown in Table 3. All results are from the point of the match player.

Table 3. Performance of match strategy vs money play strategy in 10000 5-point matches

Variant	Match Wins	Diff. moves	Games WS	Games WD	Games LS	Games LD	Total game points
Portes	5144±98	7.1%	22937	7094	19558	9066	-565
Plakoto	5103±98	4.6%	15994	10627	15238	11007	-4
Fevga	5067±98	5.3%	28395	4453	27358	5401	-635

The performance of the match strategy is better than the money-play strategy in all games, in terms of matches won by the match player, although the total points won by the match player is less than the points won by the money player. In other words, the match player is able to win the points when they are more important in order to win the current match. This observation is clearer in Portes and Plakoto, and less significant in Fevga, due to the low gammon rate of Fevga that does not give many opportunities for the players to take justified risks for a gammon. We also kept counters whether the money player would play the same move with the match strategy in a non trivial decision (number of possible moves > 1) when it was the turn of the match player (column Diff. moves). As it can be seen in this column the two strategies differ very slightly and this can be an explanation why the match strategy is only better by a small margin.

Finally, we also measured the result of each game (columns: WS, WD, LS, LD) and the total game points from the point of the match player. Interestingly, the match player wins more single games and less double games in all three variants. This can be explained with the following reasoning: when the match player is ahead on the score, it will play more conservatively trying to keep its lead and not take unnecessary chances to win a gammon that could give also winning chances to the opponent. On the other hand, when he is behind he will go more aggressively for a gammon in order to try to close the gap before it is too late. This risky strategy some times will be successful, but most of the times it will result in gammons for the opponent.

6 Conclusions and future work

In this paper we used Palamedes bot to conduct rollout experiments on the opening moves of the first player for three popular backgammon games: Portes, Plakoto and Fevga. Our findings for Portes without the double rolls are very close to those found in the literature. To the best of our knowledge, this is the first time that an analysis of the opening moves was conducted for the other two variants, Plakoto and Fevga.

Our results show that the advantage of the first player is significant in the Fevga variant, small in Plakoto and very small in Portes. The superiority of the Portes variant in this statistic was expected because Portes (and backgammon) has the advantage of a specially crafted starting position which is not present in the other variants. Another interesting result is that the gammon rates of the three games fall in completely different ranges. The smallest gammon rate is for the Fevga variant (14.27%), followed by Portes/Backgammon (26.9%), whereas Plakoto has the largest rate (at 41%).

We also showed the effect of the starting position on the statistics examined in the Plakoto variant. Changing only slightly the starting position (Tapa variant), we managed to lower significantly the gammon rate and the advantage of the first player, making Tapa the most "fair" backgammon variant examined so far. It would be interesting to try the opposite procedure in the backgammon/Portes variant: what would be the gammon rate and equity of a variant with the same rules as backgammon but a starting position where all starting checkers are placed in the player's first point? If the results of our Plakoto/Tapa experiments are any indication, we suspect that we would see an increase in both of these measurements. We could have tried out an experiment using the Portes NN in this variant. However, unlike the Plakoto/Tapa case, here the change of the starting position is significant, so we feel that the Portes NN will not generalize well. A new NN-based evaluation function should be self-trained, but as this is not trivial, we leave it for future work.

Finally, as a practical application, we used the computed gammon rates to construct a match strategy that outperformed our previous money play strategy when playing 5-point matches in Portes and Plakoto. In the future we plan to extend this method in matches where the starting player of the game is the one that wins the previous game, and in matches that consist of different game types like a Tavli match.

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APPENDIX A: Precomputed Match Winning Chance (MWC) tables for Tavli games

The following tables are calculated according to Section 5 and equation (2).

Table 4. MWC (%) for player A on Portes variant

A away		**MATCH WINNING CHANCE (MWC)**								
	B away	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
1		50.00	68.28	81.68	89.04	93.53	96.16	97.73	98.65	99.20
2		31.73	50.00	65.85	76.78	84.56	89.83	93.37	95.72	97.25
3		18.32	34.15	50.00	62.91	73.20	80.98	86.72	90.84	93.75
4		10.96	23.22	37.09	50.00	61.39	70.85	78.41	84.26	88.69
5		6.47	15.44	26.80	38.61	50.00	60.25	69.07	76.36	82.23
6		3.84	10.17	19.02	29.15	39.75	50.00	59.41	67.68	74.71
7		2.27	6.63	13.28	21.59	30.93	40.59	50.00	58.74	66.56
8		1.35	4.28	9.16	15.74	23.64	32.32	41.26	50.00	58.20
9		0.80	2.75	6.25	11.31	17.77	25.29	33.44	41.80	50.00

Table 5. MWC (%) for player A on Plakoto variant

A away		**MATCH WINNING CHANCE (MWC)**								
	B away	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
1		50.00	64.88	79.43	86.77	91.90	94.91	96.85	98.03	98.78
2		35.12	50.00	65.87	75.78	83.47	88.67	92.34	94.84	96.55
3		20.57	34.13	50.00	61.90	71.98	79.55	85.33	89.56	92.65
4		13.23	24.22	38.10	50.00	60.91	69.87	77.20	82.97	87.43
5		8.10	16.53	28.02	39.09	50.00	59.69	68.13	75.17	80.93
6		5.09	11.33	20.45	30.13	40.31	50.00	58.94	66.82	73.60
7		3.15	7.66	14.67	22.80	31.87	41.06	50.00	58.29	65.75
8		1.97	5.16	10.44	17.03	24.83	33.18	41.71	50.00	57.79
9		1.22	3.45	7.35	12.57	19.07	26.40	34.25	42.21	50.00

Table 6. MWC (%) for player A on Fevga variant

A away		**MATCH WINNING CHANCE (MWC)**								
	B away	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
1		50.00	71.43	84.18	91.18	95.09	97.27	98.48	99.15	99.53
2		28.57	50.00	66.69	78.37	86.25	91.39	94.68	96.74	98.02
3		15.82	33.31	50.00	63.91	74.72	82.70	88.39	92.33	95.00
4		8.82	21.63	36.09	50.00	62.19	72.20	80.03	85.93	90.26
5		4.91	13.75	25.28	37.81	50.00	60.98	70.32	77.92	83.88
6		2.73	8.61	17.30	27.80	39.02	50.00	60.07	68.85	76.19
7		1.52	5.32	11.61	19.97	29.68	39.93	50.00	59.35	67.65
8		0.85	3.26	7.67	14.07	22.08	31.15	40.65	50.00	58.77
9		0.47	1.98	5.00	9.74	16.12	23.81	32.35	41.23	50.00

APPENDIX B: Best move of all opening rolls per variant examined

	PORTES		PLAKOTO		FEVGA	
ROLL	**BEST MOVE**	**EQ**	**BEST MOVE**	**EQ**	**BEST MOVE**	**EQ**
SINGLE ROLLS						
21	24/23 13/11	0.006	24/22 24/23	0.042	24/21	-0.030
31	8/5 6/5	0.155	24/21 24/23	0.037	24/20	0.012
41	24/23 13/9	0.002	24/20 24/23	0.070	24/19	0.086
51	24/23 13/8	0.011	24/19 24/23	0.043	24/18	0.090
61	13/7 8/7	0.108	24/18 24/23	0.097	24/17	0.194
32	24/21 13/11	0.017	24/21 24/22	0.050	24/19	0.086
42	8/4 6/4	0.110	24/20 24/22	0.065	24/18	0.090
52	24/22 13/8	0.015	24/19 24/22	0.066	24/17	0.194
62	24/18 13/11	0.017	24/18 24/22	0.106	24/16	0.259
43	24/20 13/10	0.015	24/20 24/21	0.056	24/17	0.194
53	8/3 6/3	0.059	24/19 24/21	0.039	24/16	0.259
63	24/18 13/10	0.018	24/18 24/21	0.096	24/15	0.336
54	24/20 13/8	0.029	24/19 24/20	0.073	24/15	0.336
64	8/2 6/2	0.016	24/18 24/20	0.121	24/14	0.385
65	24/18 18/13	0.072	24/18 24/19	0.117	24/13	0.440
DOUBLE ROLLS						
11	8/7 (2) 6/5(2)	0.213	24/23 (4)	0.129	24/20	0.012
22	13/11(2) 6/4(2)	0.240	24/20 24/22 (2)	0.137	24/16	0.259
33	8/5 (2) 6/3 (2)	0.259	24/18 24/21 (2)	0.187	24/15	0.336
44	24/20(2) 13/9(2)	0.348	24/16 (2)	0.247	24/16	0.259
55	13/8 (2) 8/3 (2)	0.160	24/14 24/19 (2)	0.361	24/9 24/19	0.831
66	24/18(2) 13/7(2)	0.398	24/12 (2)	0.521	24/18	0.090